Let $\bar{\mathbf{z}}^m_{t,i} = \mathbf{z} - \{z^m_{t,i}\}$ and $\bar{\mathbf{w}}^m_{t,i} = \mathbf{w} - \{w^m_{t,i}\}$.

$$
\begin{aligned}
p(z^m_{t,i} = j | \bar{\mathbf{z}}^m_{t,i}, \mathbf{w}, s^m_t) &= \frac{p(z^m_{t,i} = j, w^m_{t,i} | \bar{\mathbf{z}}^m_{t,i}, \bar{\mathbf{w}}^m_{t,i}, s^m_t)}{w^m_{t,i} | \bar{\mathbf{z}}^m_{t,i}, \bar{\mathbf{w}}^m_{t,i}, s^m_t)} \\
&\propto= p(z^m_{t,i} = j, w^m_{t,i} | \bar{\mathbf{z}}^m_{t,i}, \bar{\mathbf{w}}^m_{t,i}, s^m_t) \\
&= p(z^m_{t,i} = j | \bar{\mathbf{z}}^m_{t,i}, s^m_t) p(w^m_{t,i} | z^m_{t,i} = j, \bar{\mathbf{z}}^m_{t,i}, \bar{\mathbf{w}}^m_{t,i}) \\
&= \int_{\boldsymbol{\theta}} p(z^m_{t,i} = j | \boldsymbol{\theta}) p(\boldsymbol{\theta} | \bar{\mathbf{z}}^m_{t,i}, s^m_t) d\boldsymbol{\theta} \int_{\phi} p(w^m_{t,i} | \phi) p(\phi | \bar{\mathbf{z}}^m_{t,i}, \bar{\mathbf{w}}^m_{t,i}) d\phi \\
&= \frac{N^{MTZ}_{m,t,j} + \alpha}{\sum_{j'} \left(N^{MTZ}_{m,t,j'} + \alpha \right)} \frac{N^{ZW}_{j,w^m_{t,i}} + \beta}{\sum_{w'} \left(N^{ZW}_{j,w'} + \beta \right)} \\
&\propto \left(N^{MTZ}_{m,t,j} + \alpha \right) \frac{N^{ZW}_{j,w^m_{t,i}} + \beta}{\sum_{w'} \left(N^{ZW}_{j,w'} + \beta \right)}.
\end{aligned}
$$

$$
\begin{aligned}
p(s^m_t = c | \bar{\mathbf{s}}^m_t, \mathbf{z}) &= \frac{p(s^m_{t+1}, s^m_t = c, \mathbf{z}^m_t | \mathbf{s} - \{s^m_t, s^m_{t+1}\}, \bar{\mathbf{z}}^m_t)}{p(s^m_{t+1}, \mathbf{z}^m_t | \mathbf{s} - \{s^m_t, s^m_{t+1}\}, \bar{\mathbf{z}}^m_t)} \\
&\propto p(s^m_{t+1}, s^m_t = c, \mathbf{z}^m_t | \mathbf{s} - \{s^m_t, s^m_{t+1}\}, \bar{\mathbf{z}}^m_t) \\
&= p(\mathbf{z}^m_t | \bar{\mathbf{z}}^m_t) p(s^m_t = c | \mathbf{s} - \{s^m_t, s^m_{t+1}\}) p(s^m_{t+1} | \mathbf{s} - \{s^m_t, s^m_{t+1}\}, s^m_t = c) \\
&= \int_{\boldsymbol{\theta}} p(\mathbf{z}^m_t | \boldsymbol{\theta}) p(\boldsymbol{\theta} | \bar{\mathbf{z}}^m_t) d\boldsymbol{\theta} \int_{\boldsymbol{\pi}} p(s^m_t = c | \boldsymbol{\pi}) p(\boldsymbol{\pi} | \mathbf{s} - \{s^m_t, s^m_{t+1}\}) d\boldsymbol{\pi} \\
&\quad \times \int_{\boldsymbol{\pi}} p(s^m_{t+1} | \boldsymbol{\pi}) p(\boldsymbol{\pi} | \mathbf{s} - \{s^m_t, s^m_{t+1}\}, s^m_t = c) \\
&= \left(\prod_j \frac{\Gamma\left(N^{SZ}_{c,j} + \alpha + N^{MTZ}_{m,t,j} \right)}{\Gamma\left(N^{SZ}_{c,j} + \alpha \right)} \frac{\Gamma\left(\sum_{j'} (N^{SZ}_{c,j'} + \alpha) \right)}{\Gamma\left(\sum_{j'} (N^{SZ}_{c,j'} + \alpha) + |\mathbf{z}^m_t| \right)} \right)^{(1)} \\
&\quad \times \left(\frac{N^{SS}_{s^m_{t-1},c} + \gamma}{\sum_{c'} \left(N^{SS}_{s^m_{t-1},c'} + \gamma \right)} \right)^{(2)} \left(\frac{N^{SS}_{c,s^m_{t+1}} + \mathbf{1}(s^m_{t-1} = c = s^m_{t+1}) + \gamma}{\sum_{c'} \left(N^{SS}_{c',s^m_{t+1}} + \mathbf{1}(s^m_{t-1} = c' = s^m_{t+1}) + \gamma \right)} \right)^{(3)}.
\end{aligned}
$$

(1) can be interpreted in two ways. First, it is the expectation of $\mathbb{E}_{\boldsymbol{\theta} \sim \mathrm{Dir}(\boldsymbol{\alpha} + \bar{\mathbf{z}}^m_t)} [\mathbf{z}^m_t | \boldsymbol{\theta}]$, where $\mathrm{Dir}(\boldsymbol{\alpha} + \bar{\mathbf{z}}^m_t)$ is a Dirichlet distribution whose parameter is $\boldsymbol{\alpha}$ added by counter $\bar{\mathbf{z}}^m_t$. Second, instead of thinking about the probability of $\mathbf{z}^m_t$ being drawn all at once, we can think of sequential draws of each element of $\mathbf{z}^m_t$. That is, if $\mathbf{z}^m_t = [z^m_{t,1}, \cdots, z^m_{t,k}]$, the probability of sequentially drawing $\mathbf{z}^m_t$ is

$$
\frac{N^{SZ}_{c,z^m_{t,1}} + \alpha}{\sum_j' \left(N^{SZ}_{c,j'} + \alpha \right)} \frac{N^{SZ}_{c,z^m_{t,2}} + \alpha}{\sum_j' \left(N^{SZ}_{c,j'} + \alpha \right)} \cdots \frac{N^{SZ}_{c,z^m_{t,k}} + \alpha}{\sum_j' \left(N^{SZ}_{c,j'} + \alpha \right)}.
$$

This is a slight abuse of notation. N^{SZ} in the $(i+1)$th term is actually N^{SZ} in the ith term after $N^{SZ}_{c,z^m_{t,i}}$ is increased by 1. This is clear if we think about $p(\mathbf{z}^m_t|\bar{\mathbf{z}}^m_t) = p(z^m_{t,1}|\bar{\mathbf{z}}^m_t)p(z^m_{t,2}|z^m_{t,1}, \bar{\mathbf{z}}^m_t)\cdots p(z^m_{t,k}|z^m_{t,1}, \cdots, z^m_{t,k-1}, \bar{\mathbf{z}}^m_t)$.

(2) and (3) can also be thought of as two consecutive draws of s^m_t and s^m_{t+1} and can be derived in the same way as above.